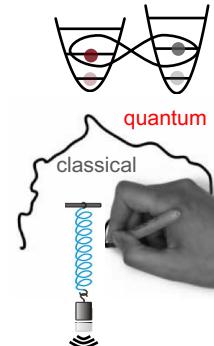


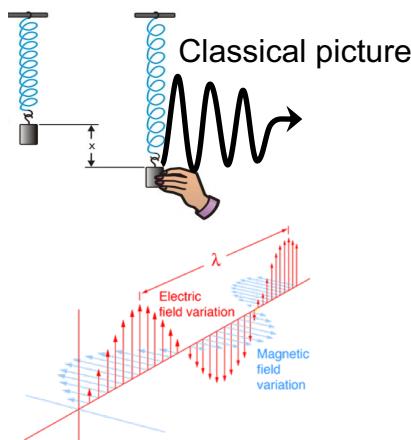
Using Light to probe Mechanical Oscillators in the Quantum Regime



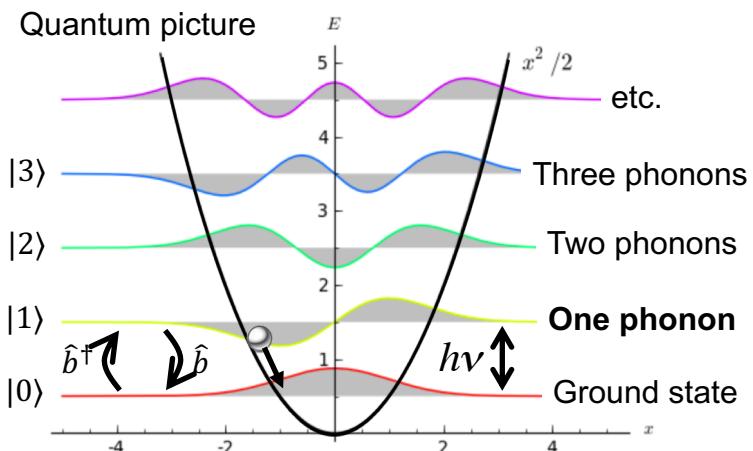
EPFL

 Christophe Galland
chris.galland@epfl.ch

The Quantized Harmonic Oscillator

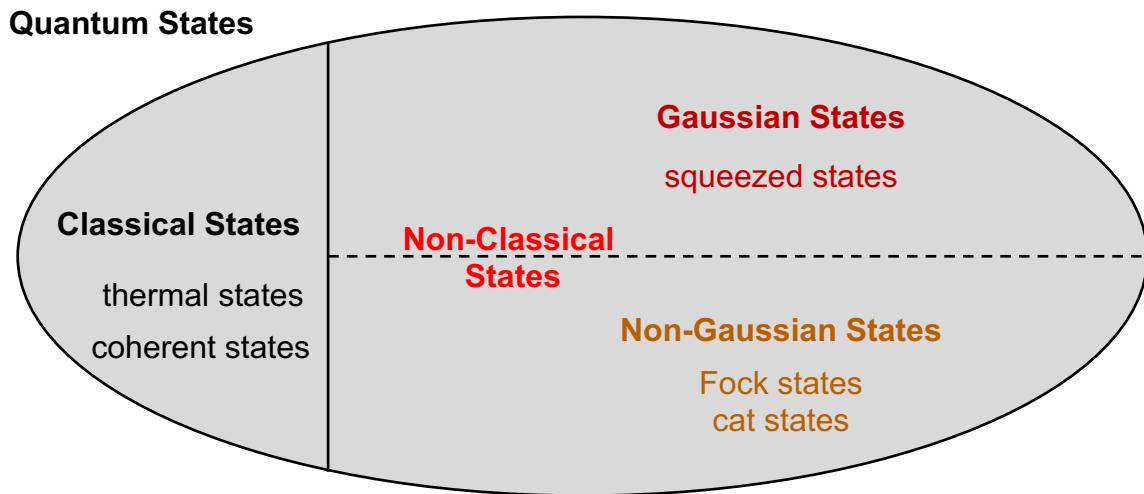


A **photon** is a quantum of electromagnetic energy



A **phonon** is a quantum of vibrational energy

Non-Classical States



NB: non-Gaussian classical states are possible <https://arxiv.org/pdf/1411.5648.pdf>

Christophe Galland, October 2019 3

Coherent State vs. Number State Representation

Glauber-Sudarshan P representation:

$$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|d^2\alpha$$

In the Fock state basis:

$$|\Psi\rangle = \sum_n c_n |n\rangle$$

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} |\alpha\rangle$$

$$\beta = \sum_{n,m} \alpha_{nm} |n\rangle\langle m|$$

Coherent state:

$$\rho = |\alpha_0\rangle\langle\alpha_0|,$$

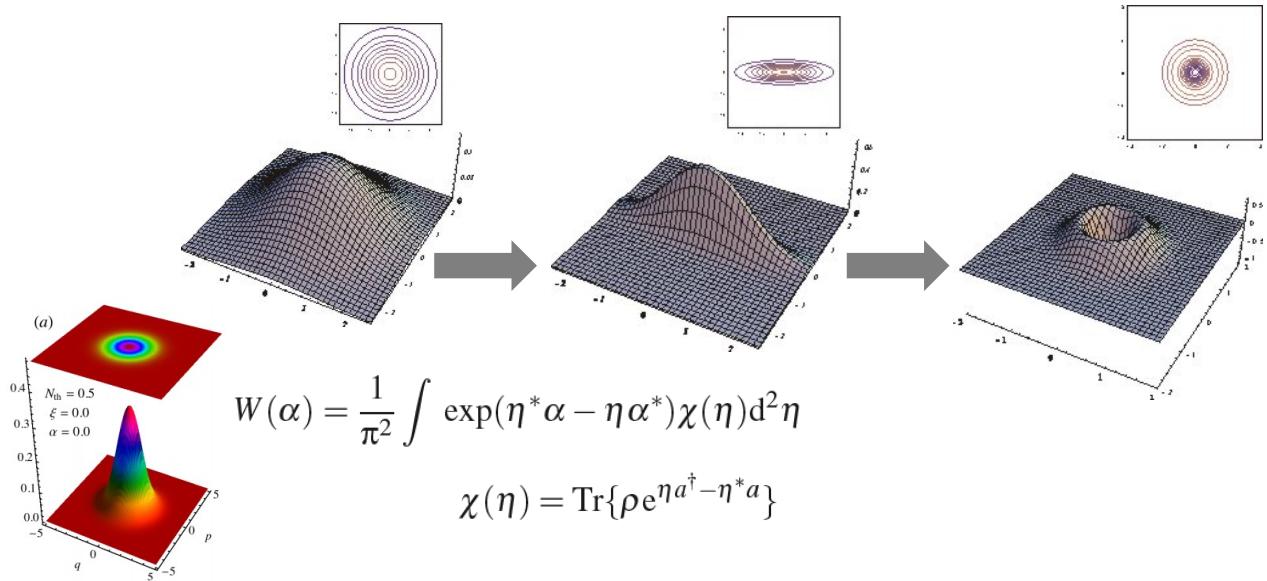
$$P(\alpha) = \delta^{(2)}(\alpha - \alpha_0)$$

Thermal state:

$$\rho = \sum P_n |n\rangle\langle n|$$

$$P(\alpha) = \frac{1}{\pi\bar{n}} e^{-|\alpha|^2/\bar{n}}$$

Wigner Function



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The photodetection probability in term of the creation/annihilation operators

$$\begin{aligned} \hat{a}^\dagger, \hat{a} &\rightsquigarrow \begin{matrix} -|e\rangle \\ \hbar\omega \hat{\sigma}_+ (\downarrow) \hat{\sigma}_- \\ -|g\rangle \end{matrix} \Rightarrow \hat{H}_{\text{int}} \propto \underbrace{\hat{a}^\dagger \hat{\sigma}_-}_{\text{emission}} + \underbrace{\hat{a} \hat{\sigma}_+}_{\text{detection / absorption.}} \\ \hat{a} &\sim e^{-i\omega t} \end{aligned}$$

$$\begin{aligned} \hat{a}^\dagger &\sim e^{+i\omega t} \quad (\text{single photon}) \quad \text{Probability of detection : } \tilde{P}_{\text{det}} \propto |\langle f | \hat{a} | i \rangle|^2 \\ &= \langle i | \hat{a}^\dagger | f \rangle \langle f | \hat{a} | i \rangle \\ P_{\text{det}} &\propto \sum_f \langle i | \hat{a}^\dagger | f \rangle \langle f | \hat{a} | i \rangle = \langle i | \hat{a}^\dagger \hat{a} | i \rangle = \langle \hat{n} \rangle \end{aligned}$$

$$\text{Two-photon detection : } \langle i | \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} | i \rangle = \langle \hat{n}^2 \rangle$$

$$\hookrightarrow \text{Normalized intensity correlation : } g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(0) \hat{a}^\dagger(\tau) \hat{a}(\tau) \hat{a}(0) \rangle}{\underbrace{\langle \hat{n}(\tau) \rangle \langle \hat{n}(0) \rangle}_{\langle \hat{n} \rangle^2}} = g^{(2)}(0) = \frac{\langle \hat{n}^2 \rangle}{\langle \hat{n} \rangle^2}$$

Classical bounds from Cauchy Schwarz inequality

$$\text{for } g \rightarrow (\int f \cdot g)^2 \leq \int f^2 \int g^2 \quad g^{(z)}(\tau) = \frac{\langle I(\tau) I(0) \rangle}{\langle I(\tau) \rangle \langle I(0) \rangle}$$

$$\hookrightarrow g^{(z)}(\tau) \leq g^{(z)}(0) \quad \text{since} \quad g^{(z)}(\tau \rightarrow \infty) = 1 \Rightarrow g^{(z)}(0) \geq 1$$

$$\text{For } l \text{ fields:} \quad g_{a,b}^{(z)} = \frac{\langle a^\dagger b^\dagger b a \rangle}{\langle a^\dagger a \rangle \langle b^\dagger b \rangle} \leq \sqrt{g_{a,a}^{(z)} \cdot g_{b,b}^{(z)}}$$

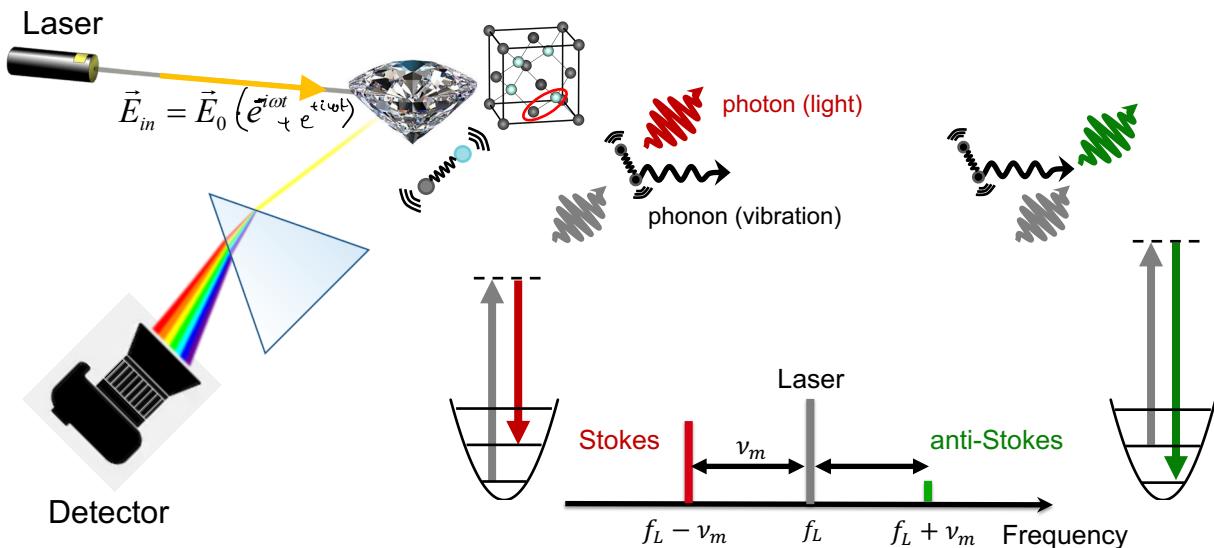
Projector for photon counting

$$I_{\text{det}} \propto \langle a^\dagger a \rangle \quad P_n = |n\rangle \langle n| \quad \rightarrow \quad P_{\text{click}} = \sum_{n \geq 1} |n\rangle \langle n| \\ = \mathbb{1} - |0\rangle \langle 0|$$

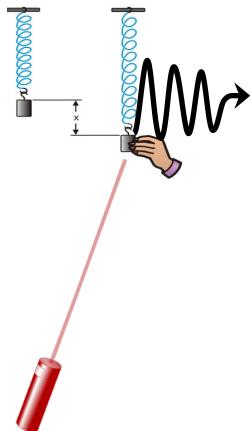
Input $|\Psi_{\text{in}}\rangle \rightarrow |P_{\text{click}} |\Psi_{\text{in}}\rangle|^2$ = probability of getting a "click".

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Using photons to create and detect single phonons



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Goals and outline

Can we use coherent (laser) light and single photon detectors to reveal non-classical properties of a mechanical oscillator?

1. The thermal state of the harmonic oscillator
2. Interaction of light with a collective oscillation
3. Two-mode squeezing interaction
4. Projective measurement and conditional Fock state
5. State readout: subpoissonian statistics
6. From single phonons to entangled states

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The Thermal State (chaotic light)

Density operator from statistical mechanics: $\hat{\rho}_{th} = \sum_n P_n |n\rangle\langle n|$ with $P_n = \frac{1}{Z} e^{-E_n/k_B T}$

$$Z = \sum_n \left(e^{-\frac{\hbar\omega}{k_B T}} \right)^n = \frac{1}{1 - e^{-\frac{\hbar\omega}{k_B T}}}$$

$$P_n = \frac{1}{Z} \tilde{p}^n$$

$E_n = \hbar\omega n$

We can show that $\tilde{p} = \frac{\bar{n}}{1+\bar{n}} \Rightarrow P_n = \left(\frac{\bar{n}}{1+\bar{n}} \right)^n$

$$\hat{\rho}_{th} = \frac{1}{1+\bar{n}} \sum_n \left(\frac{\bar{n}}{1+\bar{n}} \right)^n$$

$$\hbar\omega \ll k_B T \rightarrow \bar{n} = \frac{k_B T}{\hbar\omega}$$

Mean occupancy (phonon number) $\bar{n} = \text{Tr}(\hat{n} \hat{\rho}_{th}) = \frac{\tilde{p}}{1-\tilde{p}} = \frac{1}{(e^{\frac{\hbar\omega}{k_B T}} - 1)}$

$$\hbar\omega \gg k_B T \rightarrow \bar{n} = e^{-\frac{\hbar\omega}{k_B T}}$$

$$g_{th}^{(2)} = \frac{\langle :\hat{n}^2:\rangle}{\langle \hat{n} \rangle^2} = 2 \quad (\text{single mode thermal state})$$

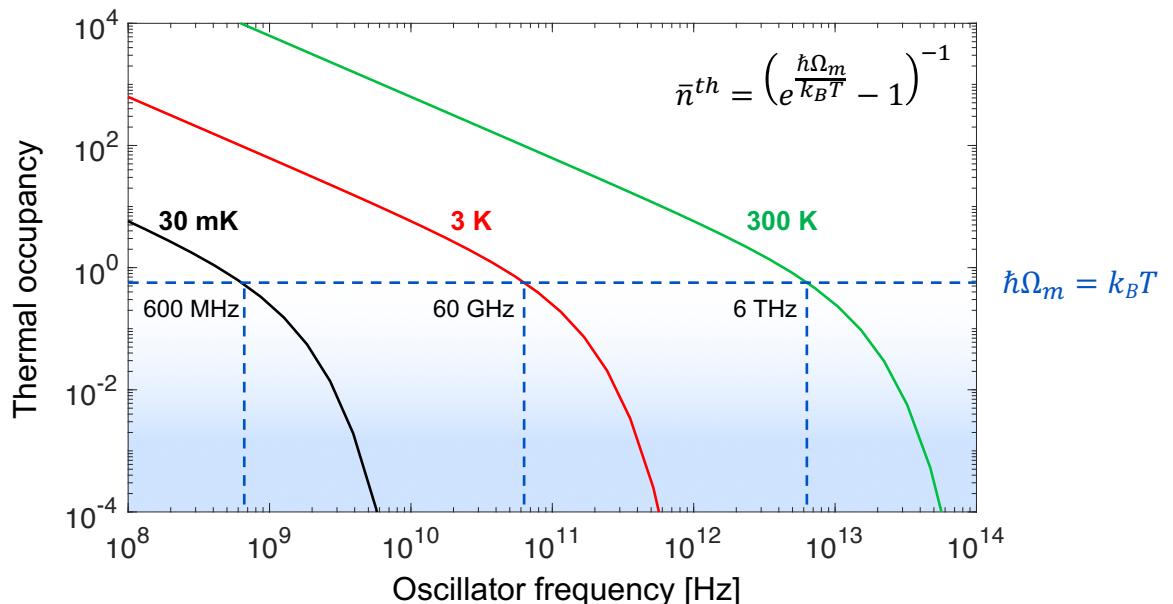
For a detector collecting the light from N modes:

$$g_{th}^{(2)} = 1 + \frac{1}{N}$$

Gerry & Knight, Ch. 2.5

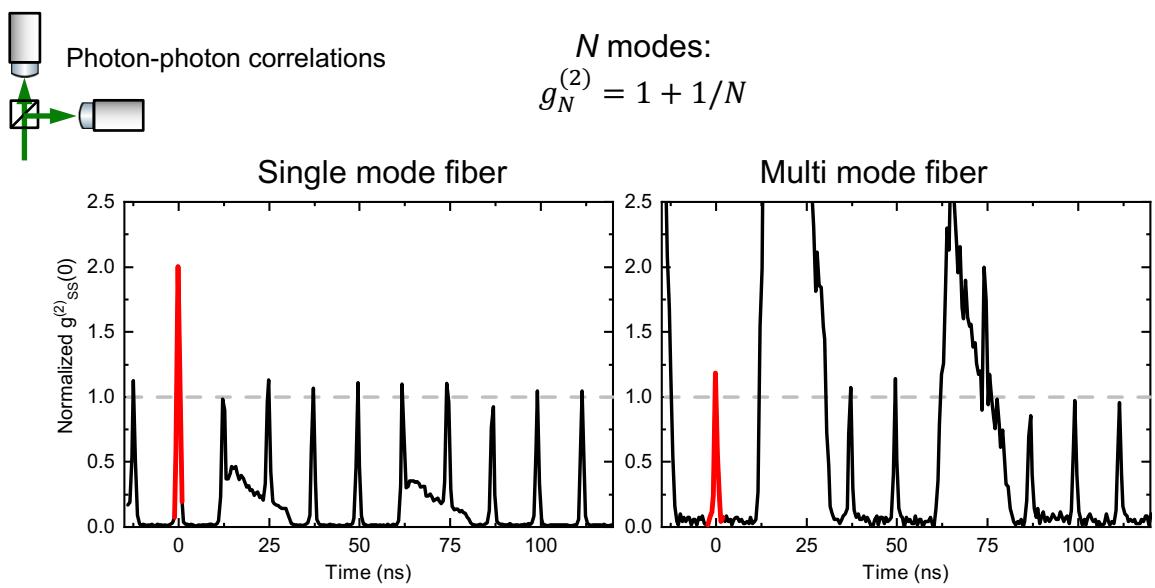
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The Bose-Einstein distribution



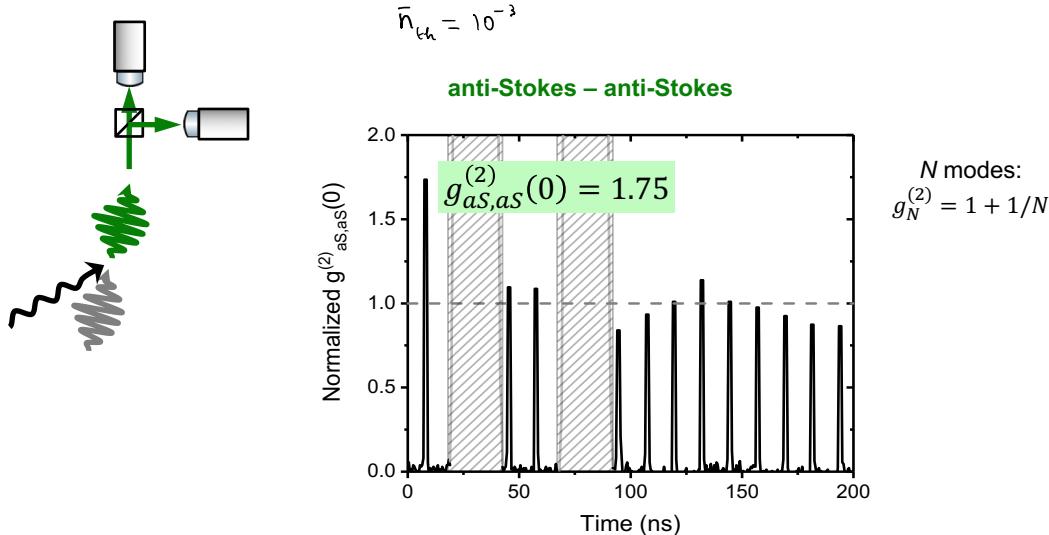
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Single vs. Multi-mode thermal statistics



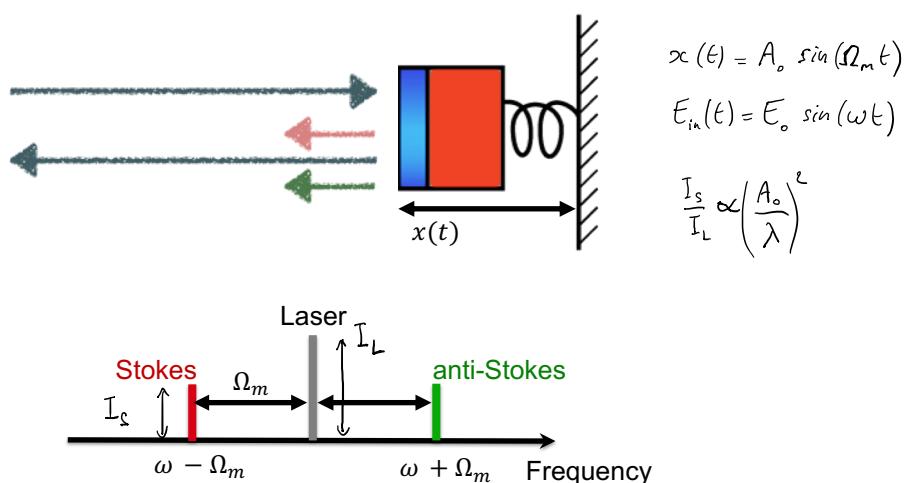
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Thermal Statistics of the Phonon Mode



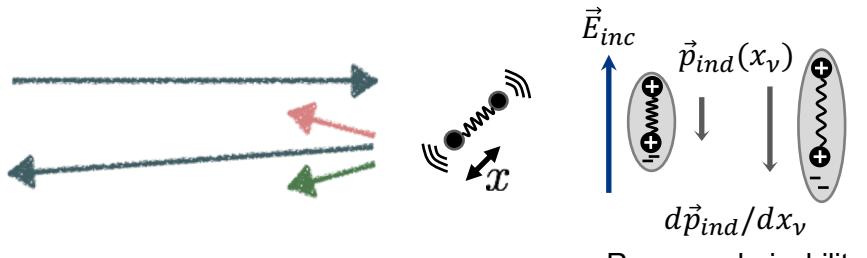
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Interaction of light with a moving boundary

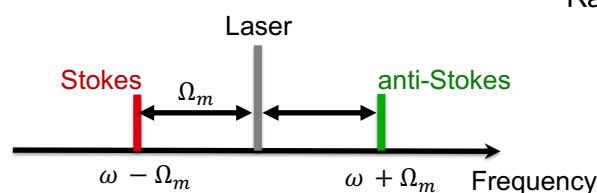


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Interaction of light with an internal vibration



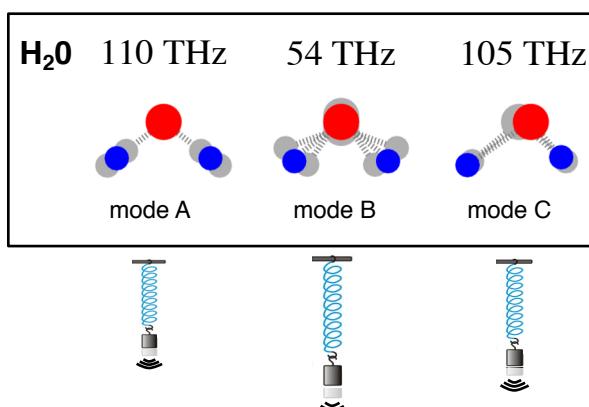
Raman spectroscopy



E.C. Le Ru and P. G. Etchegoin, *Principles of Surface-Enhanced Raman Spectroscopy and related plasmonic effects*, Elsevier , 2009

Derek A. Long, *The Raman Effect – A Unified Treatment of the Theory of Raman Scattering by Molecules*, 2002
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Coupled Oscillators: Normal Modes



Raman cross-section
 $\sim 10^{-29} \text{ cm}^2$ per C-C bond

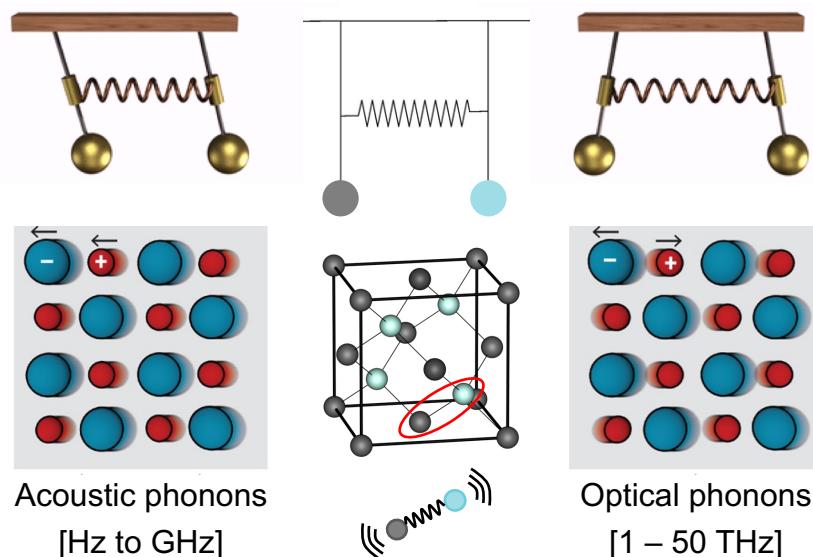


scattering probability is
 $< 10^{-21}$ per incoming photon

E.C. Le Ru and P. G. Etchegoin, *Principles of Surface-Enhanced Raman Spectroscopy and related plasmonic effects*, Elsevier , 2009

Derek A. Long, *The Raman Effect – A Unified Treatment of the Theory of Raman Scattering by Molecules*, 2002
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Coupled Oscillators: Normal Modes



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Collective Vibrational Modes

$\vec{u}(x, t)$

For each molecule : $\text{displacement (along } \vec{z} \text{)} : u(x, t)$

↳ We can expand the susceptibility : $\chi = \chi_0 + \frac{d\chi}{du} \Big|_{u=0} u(x, t) = \chi_0 + \chi_R u(x, t)$

$$\vec{D} = \epsilon_0 \vec{E}(x, t) + \vec{P} = \underbrace{\epsilon_0(1+\chi_0) \vec{E}}_{\text{in 3D: } \chi_{ijk} E_j u_k} + \underbrace{\epsilon_0 \chi_R E_z u_z}_{\rightarrow E_z(x, t) = E_L(x, t) + E_S(x, t)}$$

$$\rightarrow \epsilon_0^2 u \rightarrow \text{shift of zero point energy}$$

$$\rightarrow E_z^2 u \rightarrow \text{second order}$$

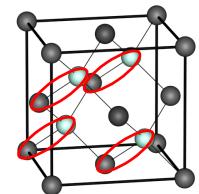
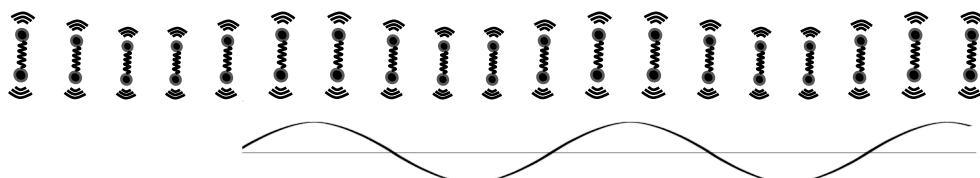
$$\mathcal{H}_{\text{tot}} = \frac{1}{2} \vec{D} \cdot \vec{E} + \frac{1}{2} \vec{B} \cdot \vec{B} = \mathcal{H}_{\text{free}} + \mathcal{H}_{\text{int}} \rightarrow \mathcal{H}_{\text{int}} = \frac{1}{2} \epsilon_0 \chi_R E_z^2(x, t) u(x, t) \Big| \rightarrow \mathcal{H}_{\text{int}} = \epsilon_0 \chi_R \epsilon_L E_z(x, t) u(x, t)$$

T. von Foerster and R. J. Glauber, Phys. Rev. A 3 1484 (1971)

Cohen-Tannoudji, Diu, Laloe Quantum mechanics vol. 1, Ch. V, complement I

Christophe Galland, October 2019 18

Collective Vibrational Modes



Quantized mechanical waves : $u(x,t) = \sum_q \sqrt{\frac{\hbar}{2L\beta\omega_L}} \left(\underbrace{\hat{b}_q e^{i(qx-\omega t)}}_{\text{mass density}} + \underbrace{h.c.}_{u^{(-)}} \right)$

Similarly for the EM field : $E(x,t) = \sum_k \sqrt{\frac{\hbar\omega_k}{2L\epsilon_0}} \left(\underbrace{i\hat{a}_k e^{i(kx-\omega t)}}_{E^{(+)}} + \underbrace{h.c.}_{E^{(-)}} \right)$

Laser : $\hat{E}_L = E_L^{(+)} + E_L^{(-)}$

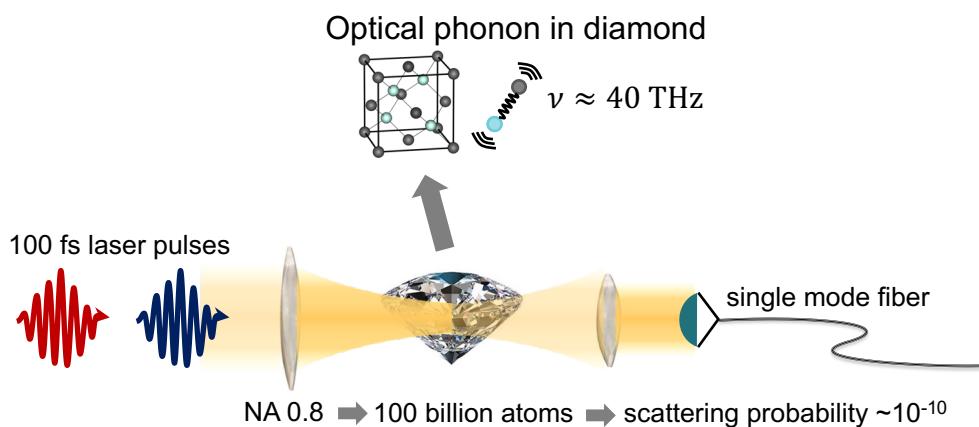
} 4 terms that contain $E_L^{(+)} E^{(+)}$
 $E_L^{(+)} E^{(-)}$
 \hookrightarrow don't conserve energy.

T. von Foerster and R. J. Glauber, *Phys. Rev. A* 3 1484 (1971)

Cohen-Tannoudji, Diu, Laloe *Quantum mechanics* vol. 1, Ch. V, complement I

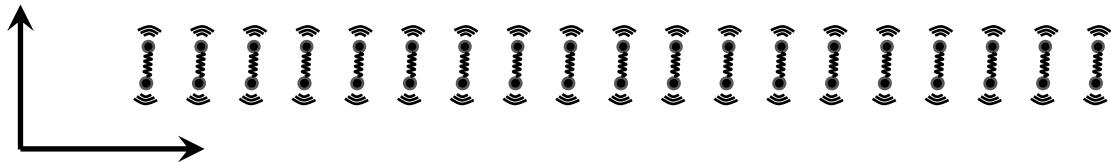
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Light to Create and Detect a Phonon



Christophe Galland, October 2019 20

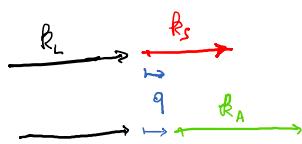
Light–vibration interaction Hamiltonian



Terms that remain in H_{int} are : $H_{\text{int}} = H_S + H_A$ where $H_S = \epsilon_0 \chi_R E^{(+)} u^{(-)} + \text{h.c.}$

$$\begin{aligned} \text{Conservation of energy : } \omega_s + \Omega &= \omega_L \\ \text{of momentum : } k_s + q_s &= k_L \end{aligned} \quad \left| \begin{array}{l} \text{Stokes} \\ \text{anti-Stokes} \\ \omega_A = \omega_L - \Omega \\ k_A = k_L + q_A \end{array} \right.$$

$$H_A = \epsilon_0 \chi E^{(+)} E^{(-)} u^{(+)} + \text{h.c.}$$



Interaction Hamiltonian :

$$\hat{H}_S = i\hbar (g_s \hat{a}_s^\dagger \hat{b}^\dagger - g_s^* \hat{a}_s \hat{b})$$

$$\hat{H}_A = i\hbar (g_A \hat{a}_A^\dagger \hat{b}^\dagger - g_A^* \hat{a}_A \hat{b})$$

T. von Foerster and R. J. Glauber, Phys. Rev. A 3 1484 (1971)

Christophe Galland, October 2019 21

Two-Mode Squeezed Vacuum

Stokes interaction : $H_S = i\hbar (g \hat{a}^\dagger \hat{b}^\dagger - g^* \hat{a} \hat{b})$

$$|\Psi_t\rangle = \hat{U}(t) |\text{vac}\rangle_{a,b} \quad \hat{U}(t) = \exp\left(\frac{tH_S}{i\hbar}\right) = \sum_n (tg \hat{a}^\dagger \hat{b}^\dagger - tg^* \hat{a} \hat{b})^n$$

Initial state is the vacuum for Stokes and vibration : $|\Psi_{\text{in}}\rangle = |0\rangle_a \otimes |0\rangle_b$

Heisenberg equation of motion :

$$i\hbar \frac{d\hat{a}}{dt} = [\hat{a}, H_S] = i\hbar g \hat{b}^\dagger \Rightarrow \frac{d\hat{a}}{dt} = g \hat{b}^\dagger \quad \rightarrow \frac{d\hat{a}}{dt} = |g|^2 \hat{a}(t) \rightarrow \hat{a}(t) = \hat{A} \cosh(|g|t) + \hat{B} \sinh(|g|t)$$

$$\frac{d\hat{b}}{dt} = g \hat{a}^\dagger \quad \left\{ \begin{array}{l} \hat{a}(t) = \hat{a}_0 \cosh(\lambda t) + \hat{b}_0^\dagger e^{i\phi} \sin(\lambda t) \\ \hat{b}(t) = \hat{b}_0 \cosh(\lambda t) + \hat{a}_0^\dagger e^{i\phi} \sin(\lambda t) \end{array} \right. \quad g = \lambda e^{i\phi}$$

$$\hat{a}(t) = \hat{U}^\dagger(t) \hat{a}_0 \hat{U}(t)$$

$$\hat{a}_0 = \hat{U}(t) \hat{a}_0^\dagger U^\dagger(t)$$

C. C. Gerry and P. L. Knight, Intro to Quantum Optics (2005) Ch. 7

Ulf Leonhardt - Measuring of Quantum State of Light (1997), Ch. 2.3

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Two-Mode Squeezed Vacuum

$\leftarrow H_s$ creates pairs ($i\hat{t}$ commutes with $\hat{n}_a - \hat{n}_b$) $\rightarrow |\Psi_t\rangle = \sum_n \alpha_n |n, n\rangle$

$$\leftarrow \hat{a} |0, 0\rangle = 0 \Rightarrow \underbrace{\hat{U}(t) \hat{a}}_0 |0, 0\rangle = 0 \Rightarrow \underbrace{\hat{U}(t) \hat{a} U^\dagger}_{0} \underbrace{U |0, 0\rangle}_{|\Psi_t\rangle} = 0$$

$|\Psi_t\rangle$ is an eigenvector of $\cosh(\lambda t) \hat{a}_0 - e^{i\theta} \sinh(\lambda t) \hat{b}_0^\dagger$

$$\hookrightarrow \sum_n \alpha_n (\sqrt{n} \cosh(\lambda t) |n-1, n\rangle + \sqrt{n+1} e^{i\theta} \sinh(\lambda t) |n, n+1\rangle) = 0$$

$$\Rightarrow \alpha_{n+1} \cosh(\lambda t) = \alpha_n e^{i\theta} \sinh(\lambda t) \Rightarrow \alpha_n = \alpha_0 (e^{i\theta} \tanh(\lambda t))^n$$

$$\sum_n |\alpha_n|^2 = 1 \Rightarrow \alpha_0 = \frac{1}{\cosh(\lambda t)} \Rightarrow |\Psi_t\rangle = \frac{1}{\cosh(\lambda t)} \sum_n e^{in\theta} \tanh^n(\lambda t) |n, n\rangle$$

$$= \sqrt{1 - \tilde{p}^2} \sum_n \tilde{p}^n |n, n\rangle$$

Marginal state (information is lost about the other mode):

$$\hat{\rho}_a = \text{tr}_b (\hat{\rho}_{\text{squeezed}}) = (1-p) \sum_n p^n |n\rangle \langle n| \quad \text{if I write } p = \frac{\tilde{n}}{1+\tilde{n}}$$

\hookrightarrow Thermal state with \tilde{n} mean occupancy.

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Outline

State of the art: Mechanical Oscillators in the Quantum Regime

Milestone 1: Preparation and Readout of a Single Phonon

Milestone 2: Bell Correlations between Light and Vibrations

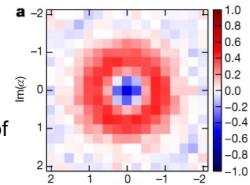
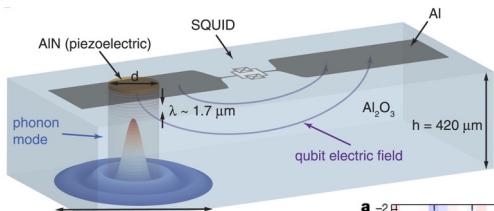
Perspectives: Synthetic Quantum Systems

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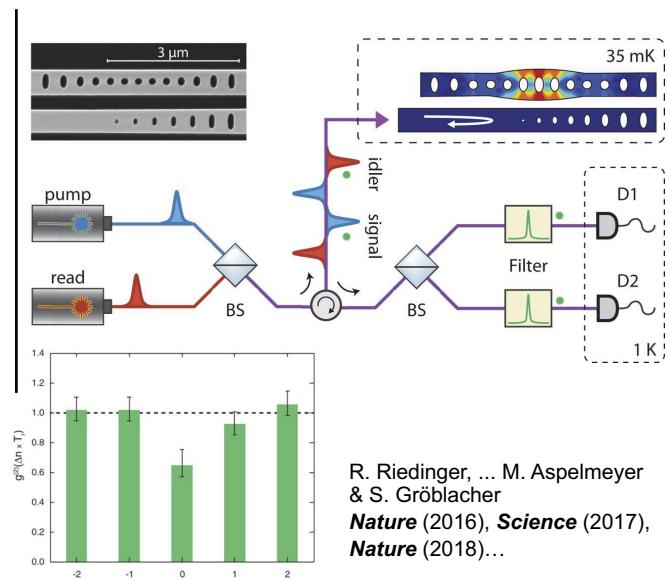
Mechanical Oscillators in the Quantum Regime

Mechanical Frequency: 5 – 6 GHz

Bath Temperature: 20 – 40 mK



Y. Chu, ... P. Rakich, R. Schoelkopf
Science (2017), *Nature* (2018)

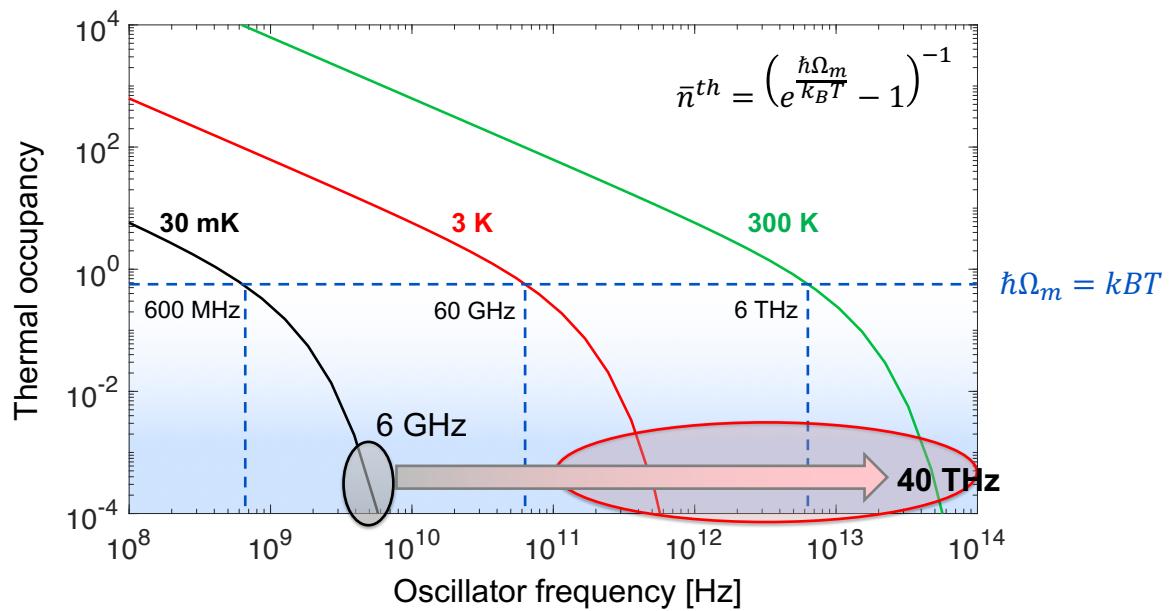


R. Riedinger, ... M. Aspelmeyer & S. Gröblacher
Nature (2016), *Science* (2017), *Nature* (2018)...

1st milestone: Phonon Fock state generation

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Mechanical Oscillators in the Quantum Regime



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Outline

State of the art: Mechanical Oscillators in the Quantum Regime

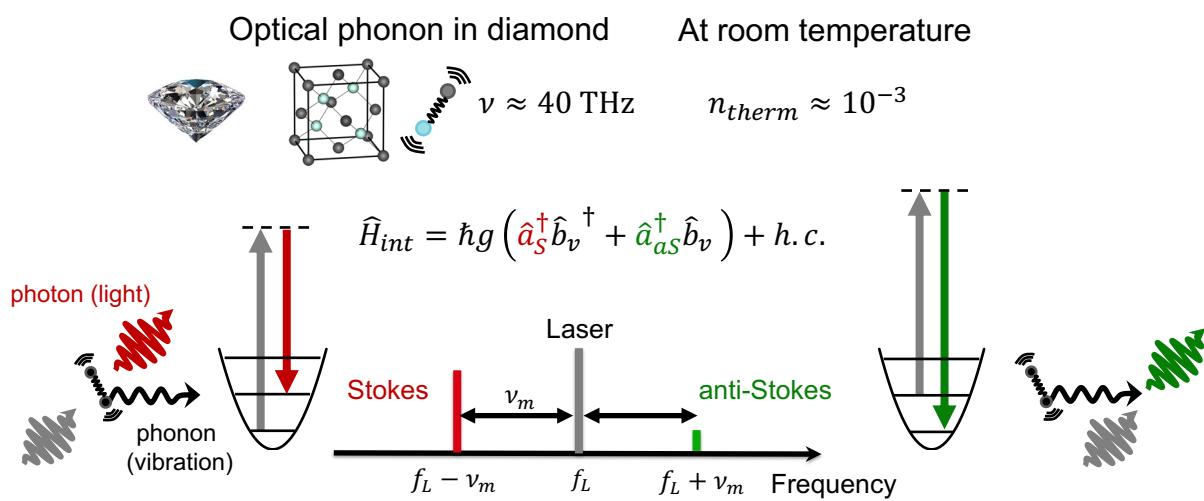
Milestone 1: Preparation and Readout of a Single Phonon

Milestone 2: Bell Correlations between Light and Vibrations

Perspectives: Synthetic Quantum Systems

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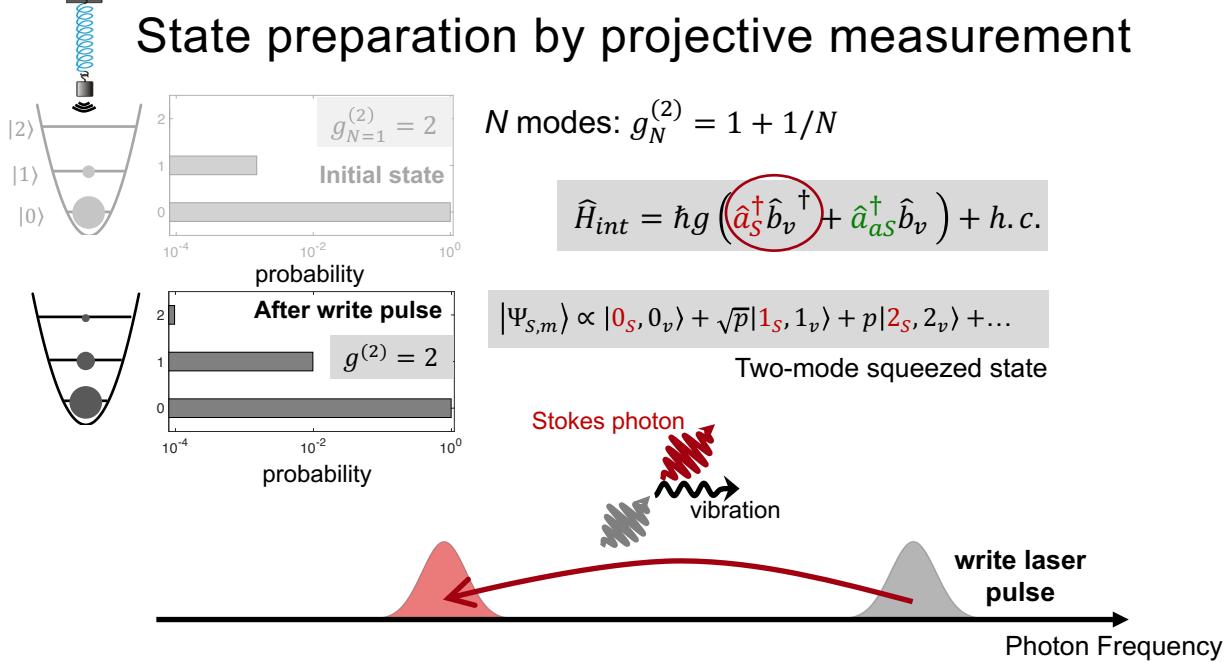
Light to Create and Detect a Phonon



K. C. Lee ..., Ian Walmsley *et al. Science* (2011) Benjamin Sussman (*PRL* 2013, *PRL* 2015, *Nat. Comm.* 2016...)
K. C. Lee, ..., Ian Walmsley *Nat. Photon.* (2012) M Kasperczyk, *Opt. Lett.* (2015); Ado Jorio (*PRL* 2017...)

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State preparation by projective measurement

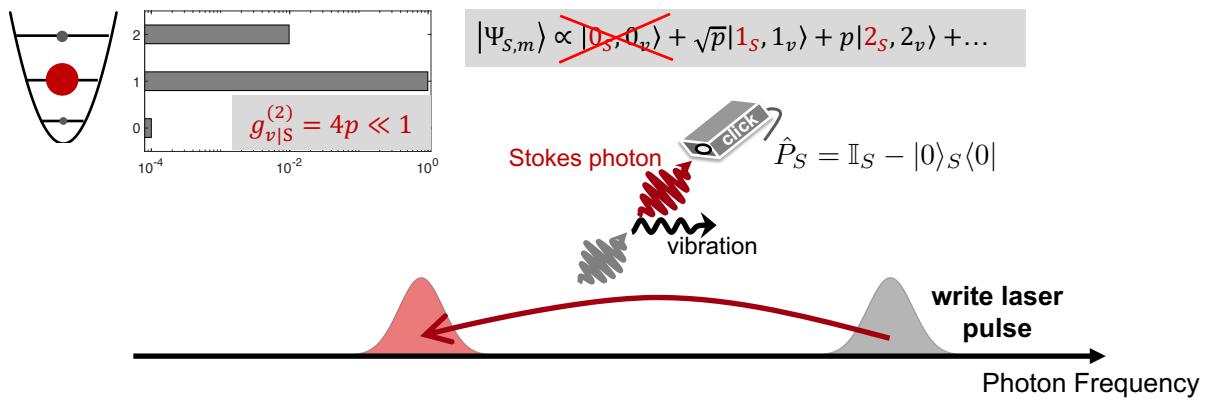


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State preparation by projective measurement

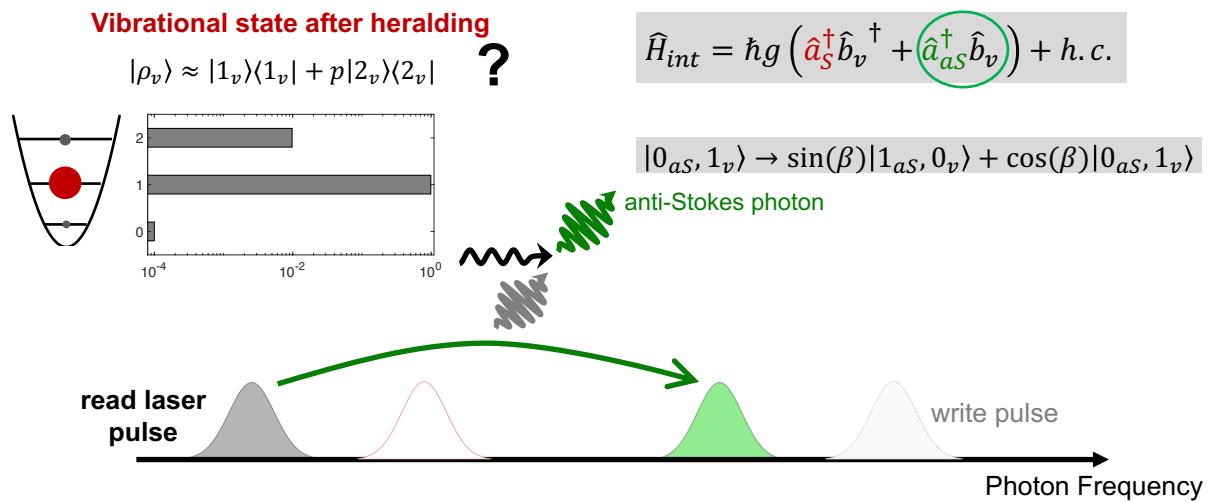
Vibrational state after heralding

$$|\rho_v\rangle \approx |1_v\rangle\langle 1_v| + p|2_v\rangle\langle 2_v| \quad ?$$

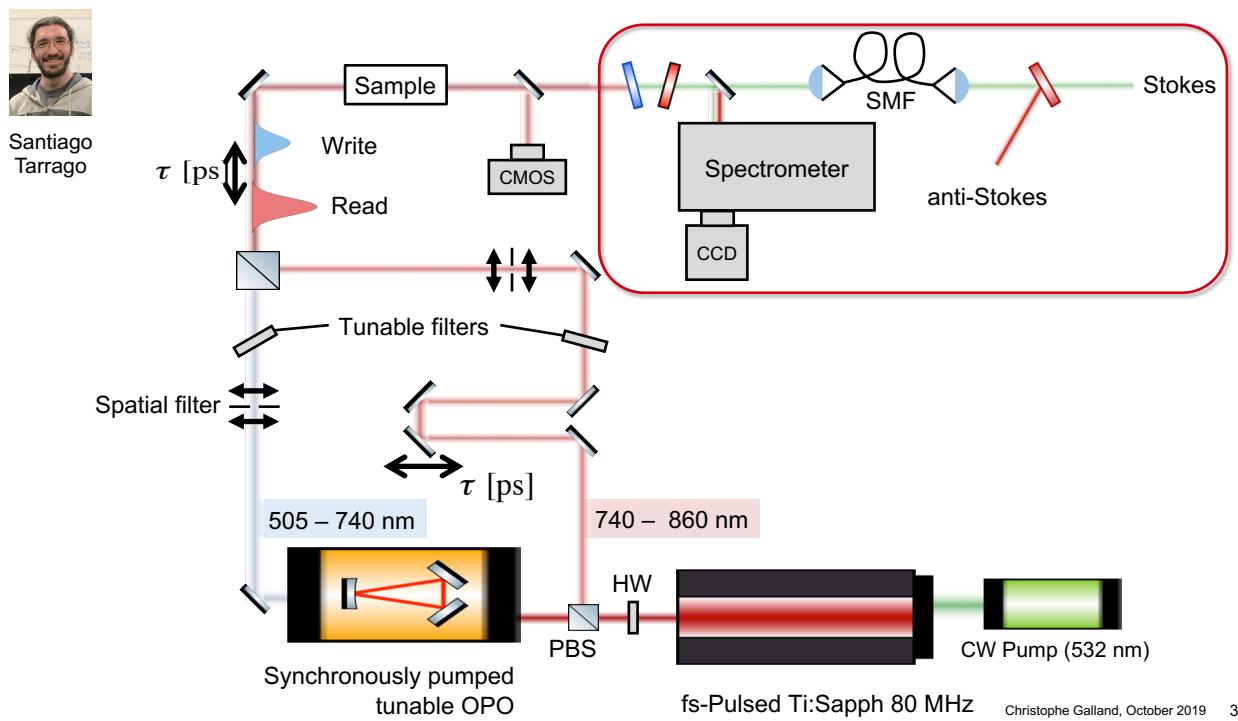


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Vibrational State read-out

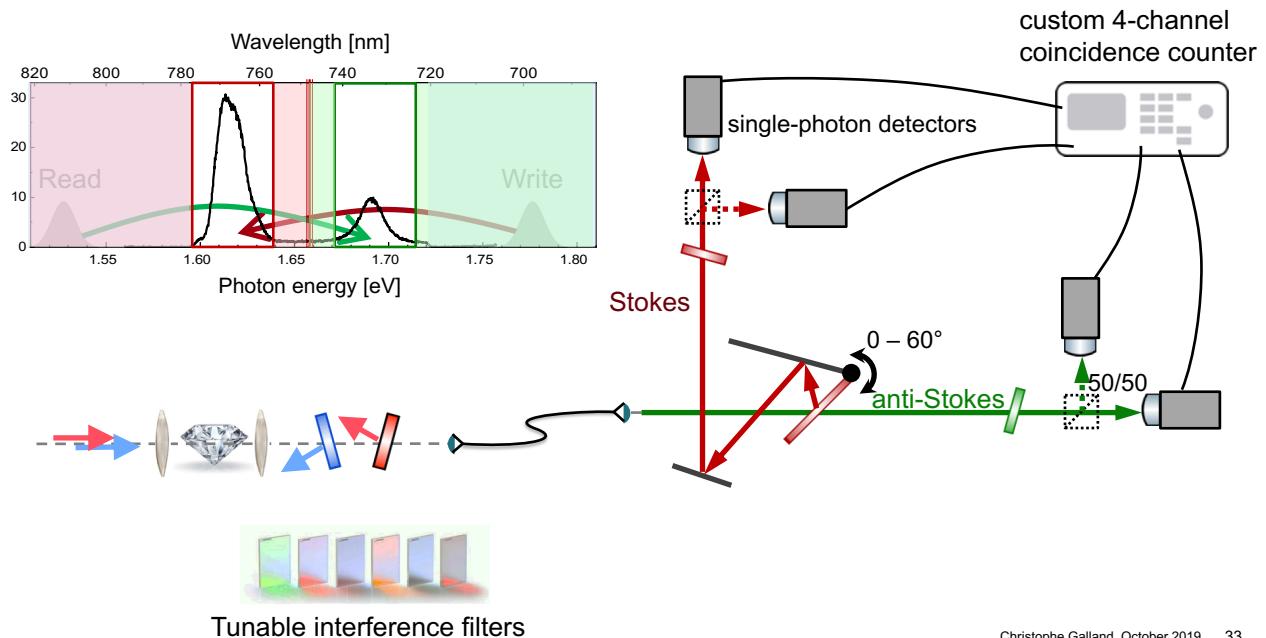


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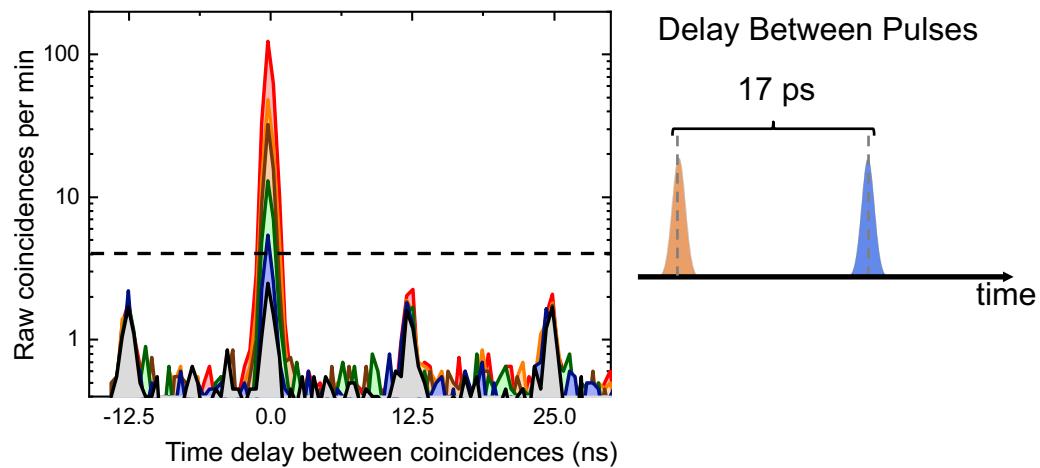
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Sideband Single-photon Detection



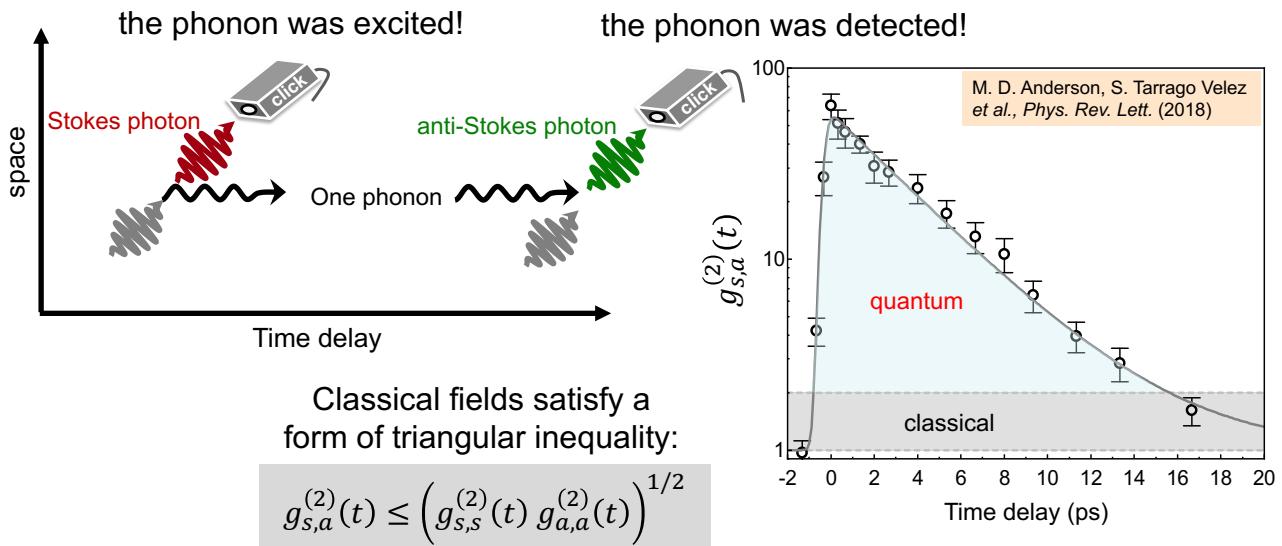
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Phonon-mediated two-color quantum correlations



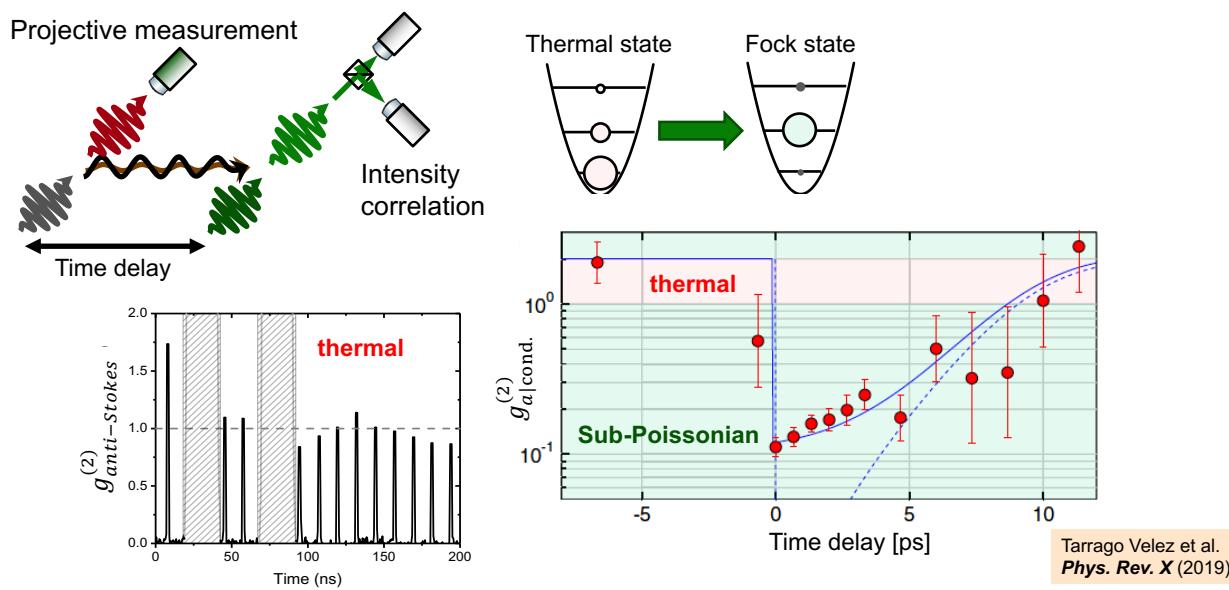
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Birth and death of a single phonon



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Phonon Fock State Preparation

Optomechanical cavity at 35 mK: $g^{(2)} = 0.6$ [S. Hong et al. Science 358, 203–206 (2017)]

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Outline

State of the art: Mechanical Oscillators in the Quantum Regime

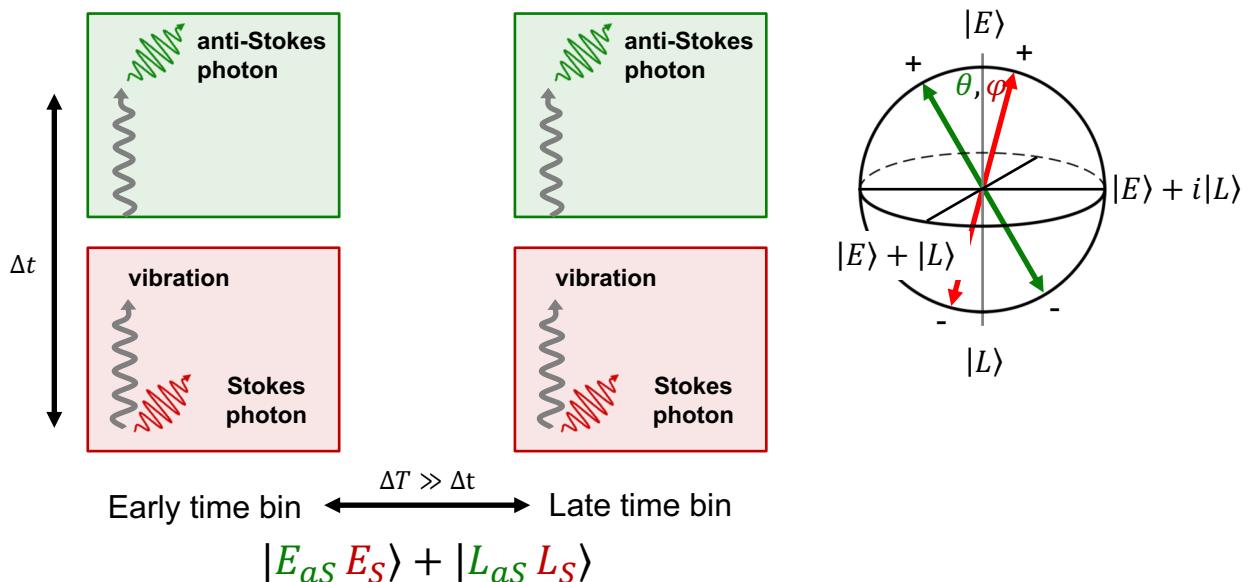
Milestone 1: Preparation and Readout of a Single Phonon

Milestone 2: Bell Correlations between Light and Vibrations

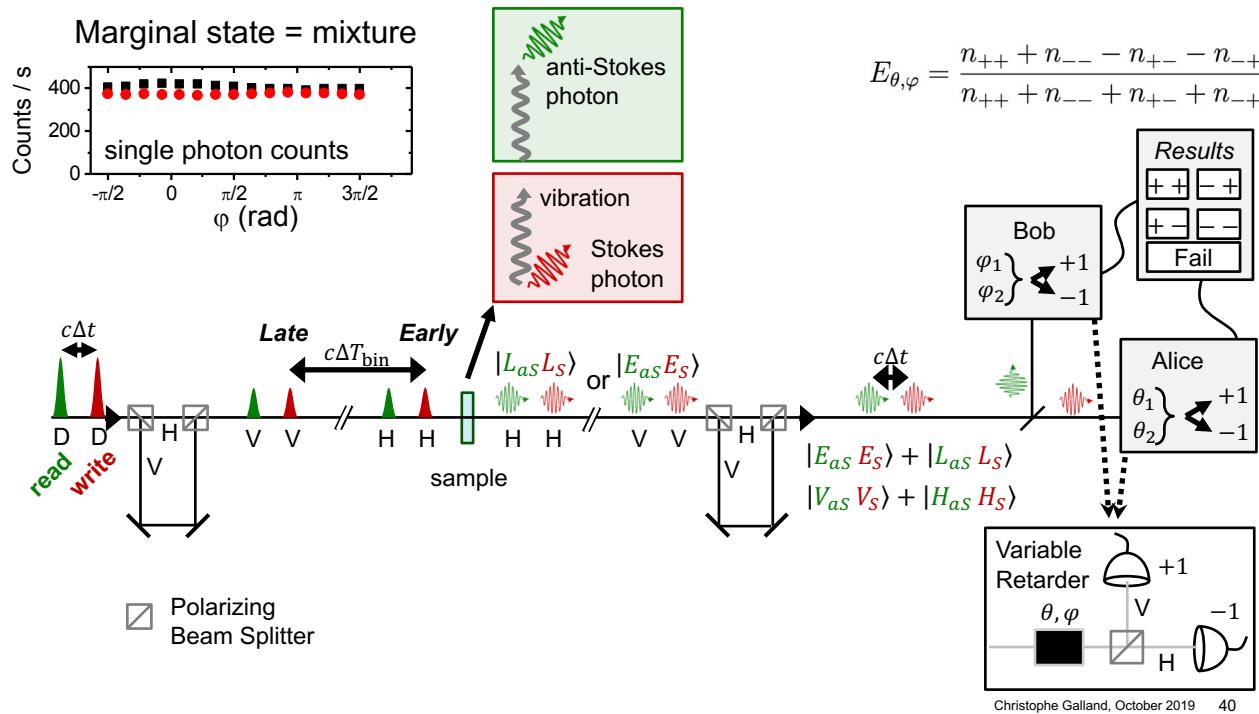
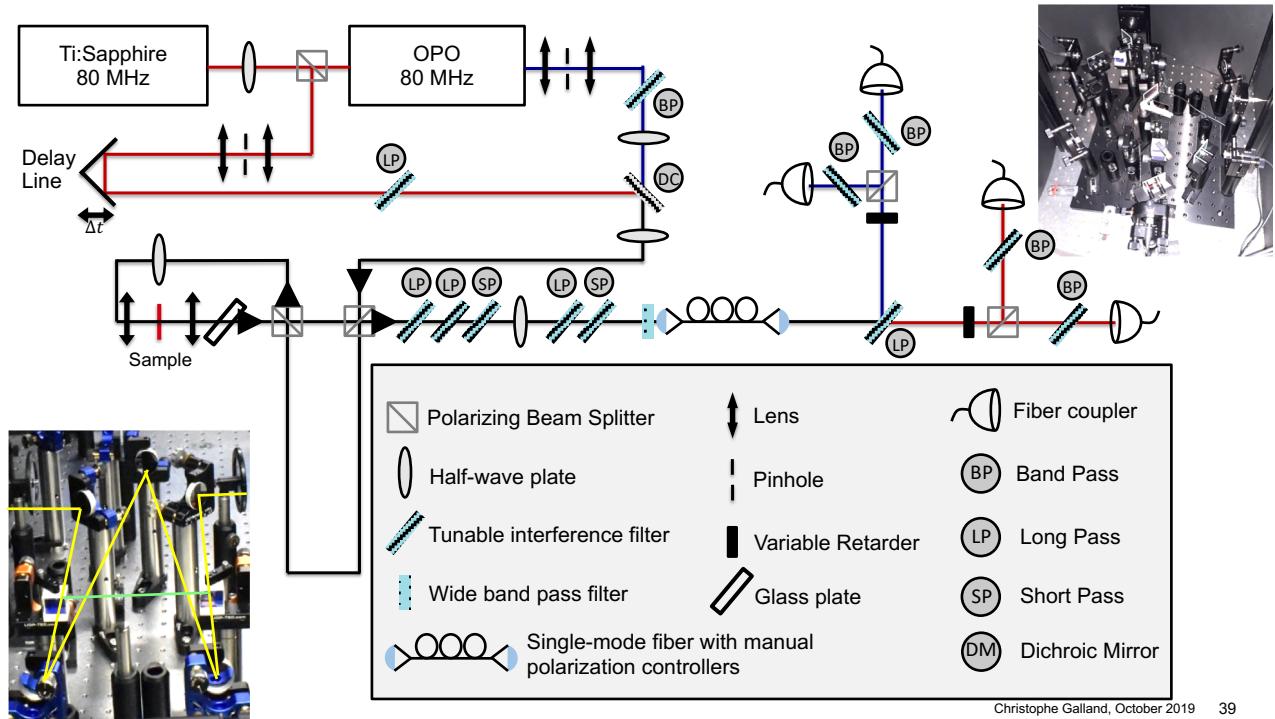
Perspectives: Synthetic Quantum Systems

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Time-bin entanglement



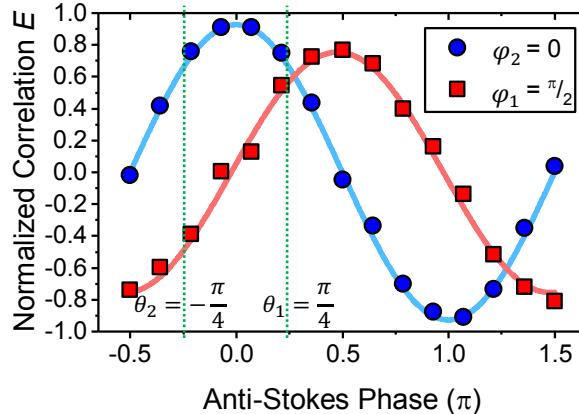
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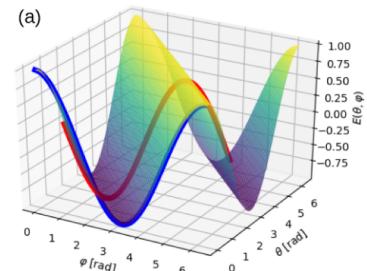
Two-photon interference and Bell correlations

write – read delay = 0.6 ps

$$S = E_{\theta_1, \varphi_1} + E_{\theta_2, \varphi_2} + E_{\theta_1, \varphi_2} - E_{\theta_2, \varphi_1} \quad S = 2.36 \pm 0.02 > 2$$



V = 93% and 75%

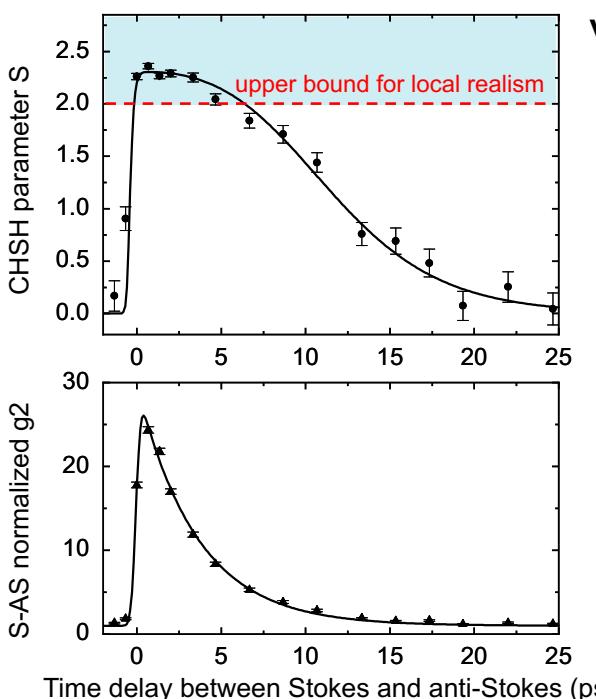


mK experiments:

R Riedinger, ... S Gröblacher, *Nature* 556, 473 (2018)

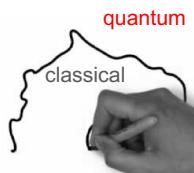
I Marinkovic, ... S Gröblacher, *Phys. Rev. Lett.* 121, 220404 (2018).

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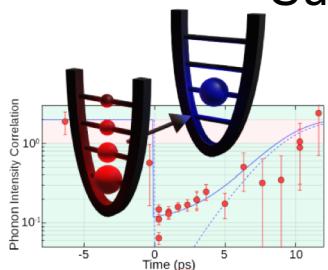
Vibrational qubit decoherence

Model inputs:
signal-to-noise (from $g^{(2)}_{S,AS}$)
imperfect time overlap (birefringence)



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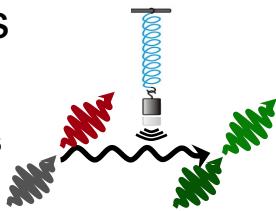
Summary of experimental results



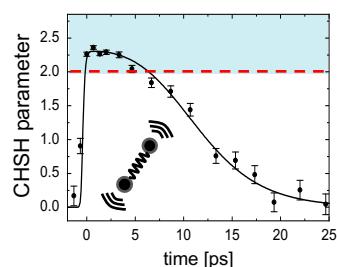
Tarrago et al. *Phys. Rev. X* (2019)

Ultrafast technique to probe
Optomechanical Quantum Correlations

**Birth and Death of a
Heralded Single Phonon**
at ambient conditions



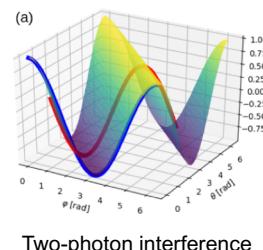
Anderson et al. *Phys. Rev. Lett.* (2018)



Tarrago et al. *to be submitted*

**Bell correlations between
light and vibration**
at room temperature

Decoherence of a vibrational qubit



Two-photon interference

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Admin and finances:
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The team @LQNO



Collaborations



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Nicolas Sangouard
Uni Basel

