Ultracoherent mechanical resonators for quantum optomechanics

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An idealized optomechanical system

- Mechanical oscillator position maps onto phase shift of light
- Radiation pressure from the light interacts with mechanical oscillator
- Cavity enhances interaction strength

M. Aspelmeyer, T. J. Kippenberg and F. Marquardt, Rev. Mod. Phys. 86, 1391
History of optomechanics

Quantum-Mechanical Radiation-Pressure Fluctuations in an Interferometer

Carlton M. Caves
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
(Received 29 January 1980)

The interferometers now being developed to detect gravitational waves work by measuring small changes in the positions of free masses. There has been a controversy whether quantum-mechanical radiation-pressure fluctuations disturb this measurement. This Letter resolves the controversy: They do.

The idea of light exerting pressure has existed since Kepler (comet tails)

Late 1960s: Seminal papers from Braginsky explore effects of radiation pressure on mechanical resonators

1980: First large interferometers (40m) built by Caltech
Rise of modern optomechanics

2000s: Techniques are developed to control mechanics with light

2010: First cooling to the quantum ground state of (macroscopic) mechanical resonators

2010s: Coupling to different systems, development of protocols for generating entanglement etc.
**Quantum technologies with optomechanics**

- **Optical RF detection**
  Bagci et al., Nature (2014)

- **Spin detection (MRFM)**
  Fischer et al., NJP (2019)

- **Quantum networking**

- **Low noise microwave-to-optical conversion**

- **Coupling to qubits**
  Chu et al., Science 359, 6373 (2017)

- **Room temperature optomechanics**

- **Quantum memory**

- **Quantum thermometry**
  Purdy et al., Science (2017)

**Room temperature optomechanics**

**Quantum thermometry**
Purdy et al., Science (2017)

**Low noise microwave-to-optical conversion**

**Quantum technologies with optomechanics**

**Spin detection (MRFM)**
Fischer et al., NJP (2019)

**Optical RF detection**
Bagci et al., Nature (2014)

**Quantum networking**

**Quantum memory**
Introduction to optomechanical interaction

\[ \hat{H}_0 = \hbar \omega_{cav}(x)\hat{a}^\dagger \hat{a} + \hbar \omega_m \hat{b}^\dagger \hat{b} \]

\[ \omega_{cav}(x) = \omega_{cav} + x \frac{\partial \omega_{cav}}{\partial x} \]

\[ \hbar \omega_{cav}(x)\hat{a}^\dagger \hat{a} \approx \hbar (\omega_{cav} - G \hat{x})\hat{a}^\dagger \hat{a}, \text{ where } G = -\frac{\partial \omega_{cav}}{\partial x} \]

\[ \hat{x} = x_{ZPF}(\hat{b} + \hat{b}^\dagger), g_0 = G x_{ZPF} \]

\[ \hat{H} = -\hbar \Delta \hat{a}^\dagger \hat{a} + \hbar \omega_m \hat{b}^\dagger \hat{b} - \hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \quad \Delta = \omega_L - \omega_{cav} \]

M. Aspelmeyer, T. J. Kippenberg and F. Marquardt, Rev. Mod. Phys. 86, 1391
A realistic optomechanical system

Figure of merit for optomechanical system (single-photon cooperativity):

\[ C_0 = \frac{4g_0^2}{\kappa \Gamma_m} \]

M. Aspelmeyer, T. J. Kippenberg and F. Marquardt, Rev. Mod. Phys. 86, 1391
Equations of motion for optomechanical systems

\[ \dot{a} = -\frac{\kappa}{2} a + i(\Delta + G\hat{x})a + \sqrt{\kappa_{ex}} a_{in} + \sqrt{\kappa_{0}} a_{vac} \]

\[ \dot{b} = \left( -\frac{\Gamma_m}{2} - i\Omega_m \right) b + i g_0 a^\dagger a + \sqrt{\Gamma_m} b_{in} \]

Cavity loss  Cavity-laser detuning  Optomechanical interaction  Cavity driving term  Vacuum fluctuations

Mechanical loss  Optomechanical interaction  Thermal bath

M. Aspelmeyer, T. J. Kippenberg and F. Marquardt, Rev. Mod. Phys. 86, 1391
Linearized Hamiltonian and equations of motion

• Consider a steady-state solution where we set $\hat{a} = \bar{a} + \delta \hat{a}$, where $\bar{a} = \sqrt{n_{\text{cav}}}$, to get:

$$H_{\text{int}} = -\hbar \sqrt{n_{\text{cav}}} g_0 (\delta \hat{a} + \delta \hat{a}^\dagger) (\hat{b} + \hat{b}^\dagger)$$

• Hamiltonian is now linear in the field operators with an effective interaction strength $g = \sqrt{n_{\text{cav}}} g_0$

• Linearized equations of motion:

$$\delta \dot{\hat{a}} = \left( -\frac{\kappa}{2} + i\Delta \right) \delta \hat{a} + ig (\hat{b} + \hat{b}^\dagger) + \sqrt{k_{\text{ex}}} \delta \hat{a}_{\text{in}} + \sqrt{k_0} \hat{a}_{\text{vac}}$$

$$\dot{\hat{b}} = \left( -\frac{\Gamma_m}{2} - i\Omega_m \right) \hat{b} + ig (\delta \hat{a} + \delta \hat{a}^\dagger) + \sqrt{\Gamma_m} \hat{b}_{\text{in}}$$

M. Aspelmeyer, T. J. Kippenberg and F. Marquardt, Rev. Mod. Phys. 86, 1391
Fundamental optomechanical interactions

- Can select different interactions with detuning

\[
\Delta = -\Omega_m \\
\Delta = 0 \\
\Delta = +\Omega_m
\]

beam-splitter (cooling)
\[
\delta \hat{a} \hat{b}^\dagger + \hat{b}^\dagger \delta \hat{a}
\]

QND
\[
\hat{x}_a \hat{x}_b
\]

squeezer (entanglement)
\[
\delta \hat{a} \hat{b}^\dagger + \hat{b} \delta \hat{a}
\]

Figure from M. Aspelmeyer, T. J. Kippenberg and F. Marquardt, Rev. Mod. Phys. 86, 1391
Dynamical backaction (cooling)

Interaction: \(-\hbar \sqrt{n_{\text{cav}}} g_0 \hat{a}^\dagger \hat{b}\)

Cooling rate: \(\Gamma_{\text{opt}} = n_{\text{cav}} C_0 \Gamma_m\)

Figure from M. Aspelmeyer, T. J. Kippenberg and F. Marquardt, Rev. Mod. Phys. 86, 1391

L. Qiu, I. Shomroni, T. J. Kippenberg (unpublished)
Two regimes of optomechanics

**Resolved sideband: \( \omega_m \gg \kappa \)**
- Can be sideband cooled to the ground state
- Coherent quantum state swaps between light and mechanics

**Unresolved sideband: \( \omega_m \ll \kappa \)**
- Can be feedback-cooled to the ground state
- Mechanical position can be measured in real time
Standard quantum limit for position measurements

Arises from tradeoff between quantum backaction from radiation pressure and photon shot noise

Photon shot noise
Radiation pressure backaction
Sum
Standard quantum limit
Quantum challenges for controlling mechanics

- High thermal occupancy
  \[ n_{th} = \frac{k_B T}{\hbar \omega_m} \gg 1 \]

- Strong coupling to environment
  \[ \Gamma_{th} \]

- Lack of intrinsic nonlinearity

- Sensitive readout required
  \[ x_{ZPF} \sim \text{fm} \]

- Hard to achieve strong coupling

M. Aspelmeyer, T. J. Kippenberg and F. Marquardt, Rev. Mod. Phys. 86, 1391
Dissipation dilution

Optical trapping

Gravitational potential

Nanomechanical resonators


González and Saulson,

Dissipation dilution in strained materials

Key insight: Stress increases mechanical quality factors **regardless of the source of loss**

Therefore: Use a high deposition stress material such as \( \text{Si}_3\text{N}_4 \)

A timeline of Si$_3$N$_4$ mechanics

Millimeter-scale membranes
Thompson et al., Nature 452, 72-75 (2008)

Trampoline membranes
Sankey et al., PRX 6, 021001 (2016)

Soft-clamping

Strain engineering
Ghadimi et al., Science 360 (2017)

Higher aspect ratios and engineered geometry has enabled 100-1000 times higher mechanical quality factor
Increasing the $Q$ with dissipation dilution

Deposition stress: 1.1 Gpa
Thickness: 20 nm
Length: 3 mm

High stress material, Si$_3$N$_4$
Dissipation dilution

Key insight: Stress increases mechanical quality factors *regardless of the source of loss*

Dissipation dilution

\[ \frac{Q}{Q_{\text{int}}} = \frac{1}{2\lambda + n^2 \pi^2 \lambda^2} \]

\[ \lambda = \frac{h}{L} \sqrt{\frac{E_0}{12\sigma}} \ll 1 \]

- \( Q_{\text{int}} \): Intrinsic \( Q \)
- \( n \): Mode number
- \( h \): Thickness
- \( L \): Length
- \( E_0 \): Young’s modulus
- \( \sigma \): Material stress

Soft clamping

Pattern a phononic crystal on the beam with a defect in the center, creating a localized mode

Dissipation dilution

\[ \frac{Q}{Q_{\text{int}}} = \frac{1}{2\lambda + n^2 \pi^2 \lambda^2} \]

\[ \lambda = \frac{h}{L} \sqrt{\frac{E_0}{12\sigma}} << 1 \]

Curvature at clamps

Distributed curvature

Vibration \( u(t) \)

Tension

Soft clamped nanobeams

Defect length ($L_d$)

More red $\rightarrow$ higher stress

Localized modes with:
10x enhanced $Q$
20x reduction in $m_{eff}$
Increasing the $Q$ with dissipation dilution

Soft clamping removes clamping losses

High stress material $\text{Si}_3\text{N}_4$
Dissipation dilution

\[
\frac{Q}{Q_{\text{int}}} = \frac{1}{2\lambda + n^2 \pi^2 \lambda^2}
\]

\[
\lambda = \frac{h}{L} \sqrt{\frac{E_0}{12\sigma}}
\]

Now, minimize \(\lambda\):
1. High aspect ratio \((h/L)\)
2. High stress \((\sigma)\)

Elastic strain engineering

Key insight: use geometry to locally increase the stress

More red → higher stress

A simple taper increases the stress in the center...
...but loss near clamps will increase, leading to lower $Q$!

Li et al., MRS Bull. 39, 108-114 (2014)
Clamp-tapering – a strain engineering technique

\[
\frac{Q}{Q_{\text{int}}} = \frac{1}{2\lambda_{\text{cl}} + n^2 \pi^2 \lambda^2}
\]

- Clamp taper increases stress at clamps \(\rightarrow\) reduced loss at clamps
- Tapering enhances \(Q\) of lower-order modes including the *fundamental mode*
- Does not require increased device size

Bereyhi et al., *Nano Lett.* 19(4), 2019
Clamp-tapering for fundamental mode $Q$ enhancement

At yield stress, factor of ~2 enhancement for Si$_3$N$_4$

Bereyhi et al., *Nano Lett.* 19(4), 2019
Elastic strain engineering with soft clamping

1. Localize the mechanical mode using soft clamping
2. Taper the beam to colocalize strain with the mechanical mode

Pitch of unit cells must be changed to maintain same bandgap throughout the beam
Increasing the $Q$ with dissipation dilution

- Soft clamping removes clamping losses
- Geometric strain engineering further increases dissipation dilution

High stress material Si$_3$N$_4$
SEMs of fabrication process

Red: Silicon nitride
Green: Silicon
Blue: Silicon pillar

Fabrication outline
1. Ebeam lithography
2. Deep etch (RIE)
3. KOH undercut
Final devices: 20 nm thick and 3-8 mm long
Strain engineered beams - bandgaps

For fixed beam length (as high as limited by fabrication):

Higher strain ➔ Smaller region of enhancement ➔ Higher frequency mode

\[ \sigma_{\text{max}} = 2 \text{ GPa} \]
\[ \sigma_{\text{max}} = 2.9 \text{ GPa} \]
\[ \sigma_{\text{max}} = 4.0 \text{ GPa} \]
More red ➔ higher stress
Strain engineered beams – quality factors

Strain engineering allows high $Q_f (> 5 \times 10^{14} \text{ Hz})$ for a wide range of frequencies

Ghadimi et al., Science 360, 2018
Unprecedented mechanical dissipation

- Highest quality factor and $Qf$ product achieved for a room temperature mechanical oscillator
- Stroboscopic measurement taken to control for photothermal anti-damping

Ghadimi et al., Science 360, 2018
How can we go beyond Si$_3$N$_4$?

$$\frac{Q}{Q_{\text{int}}} = \frac{1}{2\lambda + n^2 \pi^2 \lambda^2}$$

Two level systems

$$\sigma = E e^{i\Phi} \epsilon$$

through strain-induced relaxation


Soft-clamped devices of strained silicon

$L \approx 1 \, mm, h \approx 10 \, nm$ Aspect ratio $\approx 10^5$

Fundamental mode quality factor

$Q \approx \{5.5, 9.8\} \cdot 10^5 \, @ \, T = \{300, 6\} \, K$

$f = 279 \, kHz$

Single high order localized mode, $m_{eff} \approx 2.5 \, pg$
Integration of high-aspect-ratio beams with 1D cavities

High-Q 1D Fabry-Pérot cavity on a waveguide with photonic crystal end mirrors ($Q_{int} > 35000$)
Simulations show $g_0$ ranging from 0.1-1 MHz
Clamp-tapered beam integrated with 1D cavity

Recently measured device:

\[ \omega_m = 450 \text{ kHz} \quad \Gamma_m = 2 \text{ Hz} \]

\[ g_0 = 180 \text{ kHz} \quad \kappa = 14 \text{ GHz} \]

\[ \rightarrow C_0 \sim 2 \]
Summary

- Soft clamping, clamp-tapering and strain engineering improve dissipation dilution
- Increased $Q$ by a factor of $6.7 \times 10^5$ above the intrinsic material quality factor through dissipation dilution
- Record room temperature quality factors ($Q = 8 \times 10^8$) demonstrated in Si$_3$N$_4$ nanobeams
Why do we care about dissipation?

Dissipation ($\Gamma_m$) limits...

1. Force sensitivity
   \[ \delta F_{th} = \sqrt{4k_B T m_{\text{eff}} \Gamma_m} \]

2. Frequency stability
   \[ S_{\omega\omega}(\omega) = 2 \frac{\left\langle X^2_{\text{th}} \right\rangle}{\left\langle X^2_{\text{osc}} \right\rangle \omega^2 + (\Gamma^2 / 2)^2} \Gamma_m \]

3. Mechanical coherence
   \[ \Gamma_{\text{th}} = \frac{k_B T}{\hbar \omega_m} \Gamma_m \]


Force sensing

Mass sensing
Dissipation dilution requirements

\[ \Delta l = \frac{1}{2} [u'(x, t)]^2 \]

- \( \Delta \epsilon \)  Change in strain
- \( l \)  Segment length
- \( \Delta l \)  Segment elongation
- \( u(x) \)  Mode amplitude

Dissipation dilution requires:
1. Static strain
2. Geometric nonlinearity of strain
   Especially strong for flexural (violin) modes

Origin of dissipation dilution in stressed materials

Dissipation dilution:

\[ D_Q = \frac{Q}{Q_{\text{int}}} = 1 + \frac{\langle W_{\text{dil}}(t) \rangle}{\langle W_{\text{lossy}}(t) \rangle} \]

Increasing static strain increases the lossless potential more than the lossy potential

\[ \sigma = Ee^{-i\phi} \epsilon \]

\[ Q_{\text{int}} = 1/ \phi \]

Lossless potential:

\[ \langle W_{\text{dil}}(t) \rangle = E \int \epsilon \langle \Delta \epsilon(t) \rangle dV \]

Lossy potential:

\[ \langle W_{\text{lossy}}(t) \rangle = \frac{E}{2} \int \langle [\Delta \epsilon(t)]^2 \rangle dV \]


Curvature in clamping region of uniform beam

\[ s = \frac{x}{L} \]: the normalized beam length

Note that the characteristic length of clamping region is proportional to \( \lambda \)
Mechanical characterization setup

Can resolve Brownian motion of mechanical modes

Ringdown measurement by driving mode with a piezoelectric element

Reflected light sent to homodyne interferometer

Nanopositioners used to position chip relative to lensed fiber

<10^{-6} \text{ mbar}
Si$_3$N$_4$ nanobeam process flow

**Patterning**
Structures are patterned using electron beam lithography

**Gap definition**
Upscaled version of the first lithography followed by deep reactive ion etching

**Undercut**
Si undercut using KOH
How do we know what stress we achieve?

Expected bandgap frequency of the three geometries on the left shown as squares.

- **Blue line**: expected bandgap frequency based on unit cell length
- **Red line**: calculated including stress
Cryogenic characterization

- Enhancement factor ranging from 2-10
- Frequency shift likely due to thermal expansion mismatch
Overview of dissipation engineering techniques

Values representative of 3-mm long, 20-nm thick Si$_3$N$_4$ beam

Strained silicon stress

\[ f_1 = \sqrt{\frac{\sigma(1-\nu)}{\rho \cdot \frac{1}{2L}}} \]

Verified that stress is in fact around 1.2 GPa after fabrication.
Strained silicon fabrication process

Patterning

Encapsulation

Undercut

Selective etch

sSi

SiO₂

Al₂O₃

PECVD SiₓNᵧ

Si